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10MAT41

Fourth Semester B.E. Degree Examination, June/July 2013

Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks:100

**Note: 1. Answer FIVE full questions, selecting at least TWO questions from each part.
2. Use of Statistical tables permitted.**

PART – A

- 1 a. Use modified Euler's method to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ at $x = 0.1$ for three iterations taking $h = 0.1$. (06 Marks)
- b. Solve $\frac{dy}{dx} = x + y$, $x = 0$, $y = 1$ at $x = 0.2$ using Runge-Kutta method. Take $h = 0.2$. (07 Marks)
- c. Using Milne's predictor-corrector method find $y(0.3)$ correct to three decimals given, (07 Marks)

x	-0.1	0	0.1	0.2
y	0.908783	1.0000	1.11145	1.25253

- 2 a. Approximate y and z at $x = 0.2$ using Picard's method for the solution of $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y + z)$ with $y(0) = 1$, $z(0) = 1/2$. Perform two steps (y_1, y_2, z_1, z_2). (10 Marks)
- b. Using Runge-Kutta method solve $y'' = x(y')^2 - y^2$ at $x = 0.2$ with $x_0 = 0$, $y_0 = 1$, $z_0 = 0$ take $h = 0.2$. (10 Marks)
- 3 a. If $f(z) = u + iv$ is analytic prove that Cauchy-Reimann equations $u_x = v_y$, $u_y = -v_x$ are true. (06 Marks)
- b. If $w = z^3$ find dw/dz . (07 Marks)
- c. If the potential function is $\phi = \log \sqrt{x^2 + y^2}$. Find the stream function. (07 Marks)
- 4 a. Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = j, 0, -i$. (06 Marks)
- b. Discuss the conformal transformation $w = e^z$. Any horizontal strip of height 2π in z -plane will map what portion of w -plane. (07 Marks)
- c. State and prove Cauchy's integral formula. (07 Marks)

PART – B

- 5 a. Prove that $J_{1/2}^{(x)} = \sqrt{\frac{2}{\pi x}} \sin x$. (06 Marks)
- b. State and prove Rodrigues formula for Legendre's polynomials. (07 Marks)
- c. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomial. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 6 a. The probabilities of four persons A, B, C, D hitting targets are respectively $1/2$, $1/3$, $1/4$, $1/5$. What is the probability that target is hit by atleast one person if all hit simultaneously? (06 Marks)
- b. i) State addition law of probability for any two events A and B.
 ii) Two different digits from 1 to 9 are selected. What is the probability that the sum of the two selected digits is odd if '2' one of the digits selected. (07 Marks)
- c. Three machine A, B, C produce 50%, 30%, 20% of the items. The percentage of defective items are 3, 4, 5 respectively. If the item selected is defective what is the probability that it is from machine A? Also find the total probability that an item is defective. (07 Marks)

- 7 a. The p.d.f of x is

x	0	1	2	3	4	5	6
p(x)	k	3k	5k	7k	9k	11k	13k

Find k. Also find $p(x \geq 5)$, $p(3 < x \leq 6)$.

(06 Marks)

- b. A die is thrown 8 times. Find the probability that '3' falls,
 i) Exactly 2 times
 ii) At least once
 iii) At the most 7 times. (07 Marks)
- c. In a certain town the duration of shower has mean 5 minutes. What is the probability that shower will last for i) 10 minutes or more; ii) less than 10minutes; iii) between 10 and 12 minutes. (07 Marks)
- 8 a. What is null hypothesis, alternative hypothesis significance level? (06 Marks)
- b. The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5. Apply student's t-distribution at 5% level of significance. ($t_{0.05}$ for 8df = 2.31). (07 Marks)
- c. In experiments on a pea breeding, the following frequencies of seeds were obtained:

Round-yellow	Wrinkled yellow	Round green	Wrinkled green	Total
315	101	108	32	556

Is the experiment is in the agreement of theory which predicts proportion of frequencies 9:3:3:1 ($\chi^2_{0.05, 3df} \approx 7.815$). (07 Marks)

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Fourth Semester B.E. Degree Examination, June/July 2013
Microcontrollers

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Compare the CPU architectures: i) CISC and RISC; ii) Von – Neumann and Harvard and iii) Microprocessor with microcontroller. (12 Marks)
- b. Interface 8051 to external ROM and RAM and explain how 8051 access them. (08 Marks)
- 2 a. Explain briefly the five addressing modes of 8051 with an example for each. (10 Marks)
- b. After reset, the contents of internal memory of 8051 with address OAH and OBH contains data 22H and 33H, respectively. Sketch the contents of internal memory from address 07H to 0BH and the value of register SP, after executing the following code:
PUSH OAH
:
MOV 81H, #OBH
POP 09H. (05 Marks)
- c. Write a subroutine which checks the content of location 20H. If it is a positive number, the subroutine finds its TWO's complement and stores it in same location and returns. (05 Marks)
- 3 a. What are assembler directives? Explain any four of them. (05 Marks)
- b. If the XTAL frequency of 8051 is 8 MHz, find the time taken to execute the following program:
MOV R2, #04
MOV R1, #06
WAIT: DJNZ R2, WAIT. (05 Marks)
- c. Write 8051 ALP which checks whether the ten numbers stored from external RAM memory address, 2000H are odd/even. The program should store accordingly OOH/FFH from internal location 30H onwards. (10 Marks)
- 4 a. Interface ADC0809 to 8051 and write an ALP to convert the analog voltage connected to second channel. Display the digital value on LEDs connected to port-0. (10 Marks)
- b. Interface 8051 to a stepper motor and write an ALP to rotate the motor first +4 steps and then -6 steps. (10 Marks)

PART – B

- 5 a. What is the difference between timer and counter operation of 8051? How to start/stop the timer/counter of 8051 when i) GATE control is not used and ii) GATE control is used. (05 Marks)
- b. Explain briefly the interrupts of 8051, indicate their vector addresses. (05 Marks)
- c. Write an ALP in 8051 which generates a square wave of frequency 10kHz on pin 1.2, using timer-1. Assume XTAL frequency as 11.0592 MHz. What is the minimum frequency that can be generated? (10 Marks)

- 6 a. Explain the functions of the pins of 9-pin RS-232 connector. (04 Marks)
b. Explain how 8051 transmits the character and receives a character serially using its UART. (06 Marks)
c. Write 8051 C program to transmit serially the message 'SWITCH ON' or 'SWITCH OFF' depending on the status of the simple switch connected to pin 1.2. Use 2400 baud rate, 1 stop bit, 8 data bits format and assume XTAL frequency as 11.0592 MHz. (10 Marks)
- 7 a. Interface an LCD display to 8051 and write an ALP to display the message 'VERY GOOD'. (10 Marks)
b. With a block schematic explain the features 8255 PPI chip and its Mode-0 operation. (06 Marks)
c. If the internal memory 20H contains AAH and 07H contains 55H, what is the content of register A and status of carry bit after executing the following code:
MOV C, 07H
MOV A, #20H
ADDC A, 07H. (04 Marks)
- 8 a. Explain the architecture of MSP430 CPU with its internal block diagram. (10 Marks)
b. Explain briefly the i) Clock system and ii) Low power modes of operation of MSP430. (10 Marks)

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Fourth Semester B.E. Degree Examination, June/July 2013
Control Systems

Time: 3 hrs.

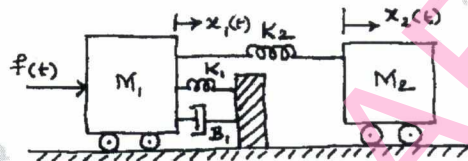
Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

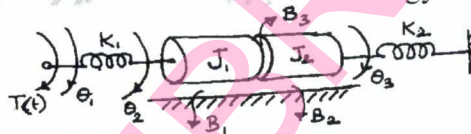
- 1 a. Compare open loop and closed loop control systems and give one practical example of each. (06 Marks)
- b. For the system shown in Fig.Q.1(b) write mechanical network and obtain its mathematical model. (06 Marks)

Fig.Q.1(b)



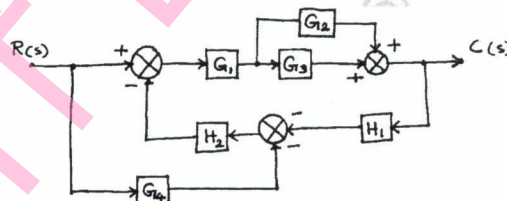
- c. For the system shown in Fig.Q.1(c) write its mechanical network and obtain mathematical model and electrical analogue based on force-current analogy. (08 Marks)

Fig.Q.1(c)



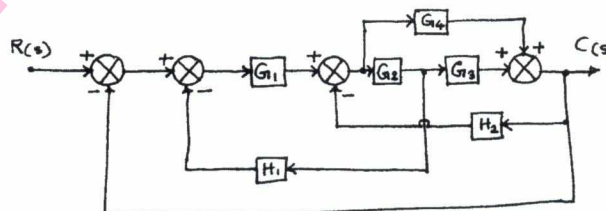
- 2 a. Define transfer function and what are its properties. (05 Marks)
- b. Obtain the transfer function for the block diagram shown in Fig.Q.2(b) using block diagram reduction method. (07 Marks)

Fig.Q.2(b)



- c. For the block diagram, given in Fig.Q.2(c) obtain over all transfer function using Mason's gain formula. (08 Marks)

Fig.Q.2(c)



- 3 a. Draw the time response curve and define time domain specifications, for second order C.S. for unit step i/p. (06 Marks)
- b. A unity feedback control system is given by an open-loop transfer function,

$$G(s) = \frac{K}{s(s+10)}. \text{ Find out:}$$

- i) The value of K for $\xi = 0.5$.
- ii) For this value of K $t_r = ?$ and $M_p = ?$ for unit step input. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.

- c. The open loop transfer function of a servo system with unity feedback is given by

$$G(s) = \frac{10}{s(0.1s + 1)}$$

Find out static error constants and obtain steady state error when subjected to an i/p of

$$r(t) = A_0 + A_1 t + \frac{A_2}{2} t^2$$

(08 Marks)

- 4 a. Explain RH stability criterion used for finding of stability of control systems. (06 Marks)
 b. Find the range of K for the system to be stable using RH criterion. (06 Marks)

$$G(s)H(s) = \frac{k(1-s)}{s(s^2 + 5s + 9)}$$

(06 Marks)

- c. Investigate the stability of the system give by characteristic equation $S^6 + 2S^5 + 8S^4 + 12S^3 + 20S^2 + 16S + 16 = 0$.

(08 Marks)

PART – B

- 5 A feedback control system has an open loop transfer function

$$G(s)H(s) = \frac{K}{s(s+3)(s^2 + 2s + 2)}$$

Draw the root locus as K varies from 0 – to - ∞.

(20 Marks)

- 6 a. Define the following terms:

- i) Resonant peak
- ii) Resonant frequency
- iii) Band width
- iv) Cut off frequency.

(04 Marks)

- b. Sketch the bode plot for the transfer function

$$\frac{300(s^2 + 2s + 4)}{s(s+10)(s+20)}$$

(13 Marks)

- c. Write a note about gain margin in brief.

(03 Marks)

- 7 a. Plot the polar plot for the transfer function given $G(s) = \frac{1}{s(Ts + 1)}$.

(06 Marks)

- b. State Nyquist stability criterion.

(02 Marks)

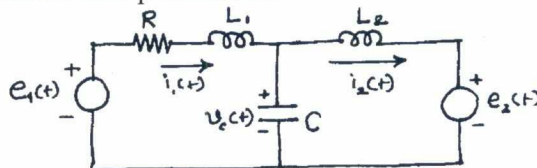
- c. Using Nyquist stability criterion verify stability of the system described below:

$$G(s)H(s) = \frac{5}{s(1-s)}$$

(12 Marks)

- 8 a. Obtain the state model for the electrical system given in Fig.Q.8(a). Take $e_1(t)$, $e_2(t)$ as i/p variables and voltage across R as o/p variables. (08 Marks)

Fig.Q.8(a)



- b. List out the properties of STM.

(05 Marks)

- c. Obtain the state transition matrix for a system matrix given by $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$.

(07 Marks)

Fourth Semester B.E. Degree Examination, June/July 2013
Signals and Systems

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Determine whether the discrete time signal $x(n) = \cos\left(\frac{\pi n}{5}\right)\sin\left(\frac{\pi n}{3}\right)$ is periodic. If periodic, find the fundamental period. (04 Marks)
- b. Determine whether the signal shown in Fig. Q1 (b) is a power signal or energy signal. Justify your answer and further determine its energy/power. (06 Marks)

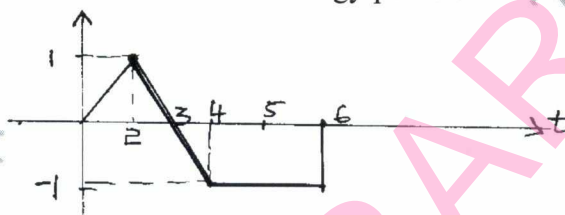


Fig. Q1 (b)

- c. Given the signal $x(n) = (6 - n)\{u(n) - u(n-6)\}$ make a sketch of $x(n)$, $y_1(n) = x(4 - n)$ and $y_2(n) = x(2n - 3)$. (04 Marks)
- d. Find and sketch the following signals and their derivatives:
 i) $x(t) = u(t) - u(t-a)$; $a > 0$ ii) $y(t) = t[u(t) - u(t-a)]$; $a > 0$. (06 Marks)
- 2 a. The impulse response of a discrete LTI system is give by, $h(n) = u(n+1) - u(n-4)$. The system is excited by the input signal $x(n) = u(n) - 2u(n-2) + u(n-4)$. Obtain the response of the system $y(n) = x(n)*h(n)$ and plot the same. (07 Marks)
- b. Given $x(t) = t$ $0 < t \leq 1$ and 0 elsewhere and $h(t) = u(t) - u(t-2)$, evaluate and sketch $y(t) = x(t)*h(t)$, $x(t)$ and $h(t)$. (07 Marks)
- c. Show that : i) $x(t)*h(t) = h(t)*x(t)$
 ii) $\{x(n)*h_1(n)\}*h_2(n) = x(n)*\{h_1(n)*h_2(n)\}$. (06 Marks)
- 3 a. Solve the difference equation, $y(n) - 3y(n-1) - 4y(n-2) = x(n)$ with $x(n) = 4^n u(n)$. Assume that the system is initially relaxed. (06 Marks)
- b. Draw the direct form I and direct form II implementations for,
 i) $y(n) - \frac{1}{2}y(n-1) - y(n-3) = 3x(n-1) + 2x(n-2)$
 ii) $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt}$ (10 Marks)
- c. Define causality. Derive the necessary and sufficient conditions for a discrete LTI system to be causal in terms of the impulse response. (04 Marks)
- 4 a. Determine the DTFS coefficients of,
 $x(n) = 1 + \sin\left\{\frac{1}{12}\pi n + \frac{3\pi}{8}\right\}$ (06 Marks)

- 4 b. Find the exponential Fourier series of the waveform shown in Fig. Q4 (b). (08 Marks)

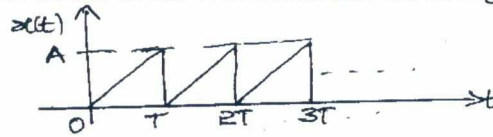


Fig. Q4 (b)

- c. Explain the Dirichlet conditions for the existence of Fourier series. (06 Marks)

PART – B

- 5 a. Find the DTFT of the signal $x(n)$ given by $x(n) = u(n) - u(n - N)$; where N is any +ve integer. Determine the magnitude and phase components and draw the magnitude spectrum for $N = 5$. (10 Marks)
- b. Determine the Fourier transform of the following signals : i) $x(t) = e^{-3t}u(t-1)$
ii) $x(t) = e^{-a|t|}$. (10 Marks)

- 6 a. Determine the frequency response and the impulse response for the system described by the differential equation,

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{d}{dt}x(t) \quad (10 \text{ Marks})$$

- b. Determine the Nyquist sampling rate and Nyquist sampling interval for,

i) $x(t) = 1 + \cos 2000\pi t + \sin 4000\pi t$ ii) $x(t) = \left[\frac{\sin(4000\pi t)}{\pi t} \right]^2$ (06 Marks)

- c. Explain briefly, the reconstruction of continuous time signals with zero order hold. (04 Marks)

- 7 a. Find the z-transform of the following and indicate the region of convergence:

i) $x(n) = a^n \cos \Omega_0(n-2)u(n-2)$

ii) $x(n) = n(n+1)a^n u(n)$ (10 Marks)

- b. Find the inverse z-transform of the following:

i) $x(z) = \frac{z^4 + z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}}; |z| > \frac{1}{2}$ by

Partial fraction expansion method.

ii) $x(z) = \frac{1 - az^{-1}}{z^{-1} - a}; z > \frac{1}{a}$ by long division method. (10 Marks)

- 8 a. A discrete LTI system is characterized by the difference equation,

$$y(n] = y(n-1) + y(n-2) + x(n-1)$$

Find the system function $H(z)$ and indicate the ROC if the system, i) Stable ii) Causal.

Also determine the unit sample response of the stable system. (10 Marks)

- b. Solve the following difference equation using the unilateral z-transform.

$$y(n] - \frac{7}{12}y(n-1) + \frac{1}{12}y(n-2) = x(n] \text{ for } n \geq 0$$

With initial conditions $y(-1) = 2, y(-2) = 4$ and $x(n] = \left(\frac{1}{5}\right)^n u(n]$. (10 Marks)

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Fourth Semester B.E. Degree Examination, June/July 2013

Fundamentals of HDL

Time: 3 hrs.

Max. Marks: 100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Describe VHDL scalar data types with an example. (08 Marks)
- b. Explain shift and rotate operators in HDL with an example. (08 Marks)
- c. Write a note on simulation and synthesis. (04 Marks)

- 2 a. Explain the execution of signal assignment statements in HDL. (05 Marks)
- b. Briefly discuss:
 - i) Constant declaration and assignment statement. (06 Marks)
 - ii) Signal declaration and assignment statement. (06 Marks)
- c. Write VHDL/Verilog code for 2×2 bit combinational array multiplier. (09 Marks)

- 3 a. Write VHDL code for 3 bit binary counter using CASE statement. (06 Marks)
- b. Explain verilog casex. Write verilog description of priority encoder using casex statement. (06 Marks)
- c. With syntax, explain the sequential statements in HDL:
 - i) IF statement
 - ii) IF as ELSE-IF
 - iii) For loop
 (08 Marks)

- 4 a. Write structural description of an VHDL SR latch using NOR gates. (08 Marks)
- b. Write the facts of structural description. (04 Marks)
- c. Write verilog code for N bit magnitude comparator using generate statement. (08 Marks)

PART – B

- 5 a. Write VHDL code to convert unsigned binary vector to integer conversion using procedure. (08 Marks)
- b. Explain the syntax of function in verilog with an example. (04 Marks)
- c. Write VHDL code for reading a string of characters from file and store in an array
Ex: COLLEGE. (08 Marks)

- 6 a. Write VHDL code for 16×8 SRAM using mixed type description. (12 Marks)
- b. Write verilog code for ALU using mixed type description. (08 Marks)

- 7 a. Write mixed language description of Master Slave D flip-flop by invoking VHDL entity from verilog module. (10 Marks)
- b. Explain the process of invoking a verilog module from VHDL module. (10 Marks)

- 8 a. With an example, explain mapping the function statement in HDL. (06 Marks)
- b. Discuss some important facts related to synthesis basics. (06 Marks)
- c. With an example, explain mapping if, if-else, case statement in HDL. Show the synthesized logic symbol and gate level diagram. (08 Marks)

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Fourth Semester B.E. Degree Examination, June/July 2013
Linear IC's and Applications

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer FIVE full questions, selecting at least TWO questions from each part.
2. Use $I_{Bmax} = 500nA$ for 741 opamp.

PART – A

1.
 - a. With a neat circuit diagram, explain the basic opamp circuit. (06 Marks)
 - b. Give definitions of the following opamp parameters and give their typical values for 741 opamp: i) CMRR; ii) Input offset voltage; iii) PSRR. (06 Marks)
 - c. Draw a neat circuit diagram for a direct coupled noninverting opamp circuit and explain the design steps. (04 Marks)
 - d. Two signals each ranging from 0.1V to 1V are to be summed. Using 741 opamp design a suitable inverting summing circuit. (04 Marks)
2.
 - a. Draw a neat circuit diagram and give design steps for a i) capacitor coupled voltage follower; ii) Capacitor coupled inverting amplifier. (08 Marks)
 - b. What is meant by setting upper cutoff in a capacitor coupled opamp? Explain how it is done in an inverting opamp. (06 Marks)
 - c. A capacitor coupled noninverting opamp is to have $A_F = 100$ and $V_O = 5V$ with $R_L = 10K\Omega$ and $f_1 = 100$ Hz. Design suitable circuit. (06 Marks)
3.
 - a. Define gain margin and phase margin and explain how they help for stability check. (06 Marks)
 - b. What is meant by frequency response compensation? Why it is required? Explain the working of a phase lag compensation network. (08 Marks)
 - c. Define: i) Slew rate effect; ii) UGB. Determine the maximum distortion free output amplitude for a voltage follower when 741 opamp is used with $f_2 = 800$ kHz and $S = 0.5$ V/ μ S. (06 Marks)
4.
 - a. What are the advantages of precision rectifier over ordinary rectifier? Explain the working of a precision halfwave rectifier. (06 Marks)
 - b. Draw the circuit of an instrumentation amplifier using opamp, explain the working and derive the expression for output. (10 Marks)
 - c. Draw the circuit and explain the working of an opamp limiter to have voltage levels $V_Z + V_D$ for negative cycle of input and $-V_{sat}$ for positive cycle of input. (04 Marks)

PART – B

5.
 - a. Write a brief note on following opamp applications:
 - i) Negative clamper using opamp.
 - ii) Log-amplifiers using opamp. (12 Marks)
 - b. Explain the working of Weinbridge oscillator using opamp. (08 Marks)

- 6 a. Explain how opamp can be used as inverting and noninverting comparator. What are the limitations? Explain how the limitations can be overcome using a Schmitt trigger. (10 Marks)
- b. An inverting Schmitt trigger circuit is to have $UTP = 0V$, $LTP = -1V$. Design a suitable circuit using bipolar opamp with $\pm 15V$ supply. (05 Marks)
- c. Design a second order active lowpass filter for a cutoff frequency of 7 kHz. (05 Marks)
- 7 a. Mention the advantages of IC voltage regulators. Draw the internal schematic for IC723 regulator and briefly explain the working. (10 Marks)
- b. Explain briefly about 78XX and 79XX series voltage regulators. (10 Marks)
- 8 a. Draw the internal schematic of 555 IC, configuring it for astable operation, explain the working. (08 Marks)
- b. Explain the working of D to A converter using R-2R network. (06 Marks)
- c. Explain the working of A to D converter using successive approximation method. (06 Marks)

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Fourth Semester B.E. Degree Examination, June/July 2013

Advanced Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. If l, m, n are the direction cosines of a line then prove that $l^2 + m^2 + n^2 = 1$. (06 Marks)
- b. Show that the direction ratios of three lines $2, 1, 1; 4, \sqrt{3} - 1, -\sqrt{3} - 1$ and $4, -\sqrt{3} - 1, \sqrt{3} - 1$ are equally inclined to one-another. (07 Marks)
- c. Find the expression for the angle between two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 . (07 Marks)
- 2 a. Find the equation of the plane passing through three points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) . (06 Marks)
- b. Find the equation of the plane through the point $(3, -3, 1)$ and its normal to the line joining the points $(3, 2, -1)$ and $(2, -1, 5)$. (07 Marks)
- c. Find the equation of the plane through $(1, -2, 2), (-3, 1, -2)$ and perpendicular to the plane $2x - y - z + 6 = 0$. (07 Marks)
- 3 a. If $\vec{a} = r \cos \theta \sin \phi \hat{i} + r \sin \theta \sin \phi \hat{j} + r \cos \theta \hat{k}$, then show that $|\vec{a}| = r$. (06 Marks)
- b. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. (07 Marks)
- c. Show that the position vectors of the vertices of a triangle $\vec{a} = 3(\sqrt{3}\hat{i} - \hat{j}), \vec{b} = 6\hat{j}, \vec{c} = 3(\sqrt{3}\hat{i} + \hat{j})$ form an isosceles triangle. (07 Marks)
- 4 a. Find the unit tangent vector to the space curve $x = \cos t^2, y = \sin t^2$ and $z = 0$. (06 Marks)
- b. Prove that $\frac{d}{dt}[\vec{F}, \vec{G}, \vec{H}] = \left[\frac{d\vec{F}}{dt}, \vec{G}, \vec{H}\right] + \left[\vec{F}, \frac{d\vec{G}}{dt}, \vec{H}\right] + \left[\vec{F}, \vec{G}, \frac{d\vec{H}}{dt}\right]$ (07 Marks)
- c. Find the tangent and normal components of its acceleration at $t = 1$ of a particle moves along the curve $\vec{r} = t^2\hat{i} - t^3\hat{j} + t^4\hat{k}$. (07 Marks)
- 5 a. Prove that $\text{div}(\vec{A} + \vec{B}) = \text{div}\vec{A} + \text{div}\vec{B}$. (06 Marks)
- b. If \vec{A} is a vector function and ϕ is a scalar function then $\nabla \times (\phi\vec{A}) = \phi(\nabla \times \vec{A}) + \nabla\phi \times \vec{A}$. (07 Marks)
- c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at $(2, -1, 2)$. (07 Marks)
- 6 a. Find $L[f(t)]$ given that $f(t) = \begin{cases} 0 & \text{for } 0 < t < 2 \\ 4 & \text{for } t > 2 \end{cases}$. (05 Marks)
- b. Find: i) $L[\cos^2 4t]$; ii) $L[\sin 2t \cos 3t]$; iii) $L\left[\frac{1 - \cos t}{t}\right]$ (15 Marks)

7 a. If $L[f(t)] = F(s)$, show that $L\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}F(s)$. (05 Marks)

b. Find: i) $L^{-1}\left[\frac{5s+1}{s^2+16}\right]$

ii) $L^{-1}\left[\frac{1}{(s+1)(s+2)(s+3)}\right]$

iii) $L^{-1}\left[\frac{s}{(s+2)^3}\right]$ (15 Marks)

8 a. Using Laplace transform solve:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}, \quad y(0) = 0 = y'(0).$$
(10 Marks)

b. Solve the system of equation using Lapalce transforms $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$. (10 Marks)
